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PREDICTION OF DROPLET SIZE FROM THE BREAKUP OF CYLINDRICAL LIQUID JETS

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Abstract—A simple equation to predict the size of droplets formed during the breakup of cylindrical liquid jets is obtained from instability analysis. This droplet-size equation applies to low-velocity, liquid-in-liquid and liquid-in-gas jets involving Newtonian or non-Newtonian fluids that follow power-law shear stress versus deformation rate relationships. The equation is tested by comparing the resultant theoretical predictions for droplet size with experimental data for seventeen Newtonian liquid systems and five power-law non-Newtonian/Newtonian liquid systems (power-law liquid jet in Newtonian liquid and Newtonian liquid jet in power-law liquid), as well as with numerical solutions to Tomotika's equation. Good agreement is observed. The present analysis demonstrates clearly the dependence of droplet size on a modified Ohnesorge number.

Key Words: liquid jet, instability, droplet size, Ohnesorge number

1. INTRODUCTION

When a liquid issues from an orifice into another immiscible fluid (either liquid or gas), droplets may form near the orifice or at the end of the jet as a result of capillary instability (Rayleigh 1945). This phenomenon is the basis for a wide range of phase-contact applications involving liquid-in-gas systems (e.g. atomization and sprinkling), and liquid-in-liquid systems (e.g. liquid–liquid extraction and emulsification). Linearized theories for jet instability and breakup have been developed by Rayleigh (1878) & Weber (1931) for liquid-in-gas systems and by Tomotika (1935) for liquid-in-liquid systems. According to Rayleigh (1878), the breakup of the jet is induced by the "most-unstable wave", which experiences the maximum growth rate in amplitude and can be determined at the early stage of jet instability. This maximum instability theory leads to the formation of uniformly-sized droplets.

Tyler (1933) was first to analyze droplets formed from the breakup of cylindrical liquid jets discharging into a gas. By applying Rayleigh's instability theory for inviscid liquid jets in a vacuum and a mass balance at the end of the jet, Tyler obtained the following relationship between droplet diameter, d, and the undisturbed jet diameter, d_j: $d = 1.91d_j$. Based on Tyler's relationship, if the undisturbed jet diameter is known, droplet size can be determined. However, in many liquid-in-liquid systems, droplet size is strongly influenced by the properties of the jet and ambient fluids, and since the jet is very short, the undisturbed jet diameter is largely a theoretical abstraction; thus, Tyler's relationship is not appropriate for liquid-in-liquid systems.

Liquid-in-liquid droplets generally are not as uniform in size as those formed in liquid-in-gas systems. Meister & Scheele (1969) found that in many liquid-in-liquid systems, at the early stage of jet instability, surface deformation of the jet is harmonic. However, at the latter stage of jet instability, higher harmonics occur on the jet surface and surface deformation of the jet becomes irregular, producing non-uniform-size droplets. In these cases, Rayleigh's maximum instability theory is not applicable. Kitamura *et al.* (1982) pointed out that the pattern of droplet formation in liquid-in-liquid systems depends on the jet velocity. (This differs significantly from liquid-in-gas systems in which the jets generally form uniformly-sized droplets in the capillary instability regime.) If the jet velocity is low, the surface disturbance wave is harmonic, producing uniformly-sized droplets. However, as the jet velocity increases, the surface disturbance wave becomes irregular, producing non-uniformly-sized droplets. Based on the experiments performed by Meister & Scheele

(1969) and Kitamura *et al.* (1982), in liquid-in-liquid systems, uniform-size droplets form only in systems with low jet velocities. Kitamura *et al.* (1982) demonstrated that droplet sizes are also uniform in liquid-in-liquid systems with low jet-versus-ambient relative velocities, and if the relative velocity is zero, droplet size does not change with changes in absolute velocity.

In low-velocity, liquid-in-liquid systems, Tomotika's linearized instability theory can be applied to the analysis of droplet formation (Kitamura *et al.* 1982). However, Tomotika's dispersion relationship (Tomotika 1935) is an implicit equation in complex form; therefore, applying that equation to the analysis of droplet formation is extremely difficult. Although several relatively simple limiting solutions to Tomotika's equation have been obtained (e.g. Meister & Scheele 1967; Lee & Flumerfelt 1981), those solutions are not applicable to many flow systems of practical importance.

Employing an integro-differential approach, Kinoshita *et al.* (1994) recently developed a simplified alternative to Tomotika's implicit, complex equation—an explicit dispersion equation applicable to both liquid-in-liquid and liquid-in-gas systems. The present, follow-on study, is undertaken to (1) develop a theoretical method based on the instability analysis by Kinoshita *et al.* to predict the size of droplets formed in the breakup of liquid jets and (2) validate the theoretical results by comparing them with experimental data and with numerical solutions to Tomotika's equation.

2. THEORY

The system of interest comprises a viscous, low-velocity liquid jet issuing into another immiscible, initially stagnant viscous fluid (liquid or gas). The flows of both the jet and ambient fluids are assumed laminar. As a result of capillary instability, the jet breaks up into discrete droplets.

At low jet velocities, jet breakup results in a train of uniform-size droplets (Meister & Scheele 1969; Kitamura *et al.* 1982; Kitamura & Takahashi 1982). In the present analysis, these droplets are assumed to be spherical, with diameter d. It is further assumed that the rate of droplet formation is constant and, over time interval Δt, N droplets are produced. The volume of the N droplets is $V_1 = N\pi d^3/6$. Over the same time interval, the volume of the jet fluid exiting the orifice is

$$V_2 = \int_{\Delta t} \frac{\pi}{4} d_0^2 u_0 \, \mathrm{d}t = \frac{\pi}{4} d_0^2 u_0 \Delta t, \qquad [1]$$

where d_0 and u_0 denote orifice diameter and bulk-mean jet velocity at the orifice, respectively. For constant jet fluid density, continuity demands that $V_1 = V_2$. Thus,

$$N \frac{\pi}{6} d^3 = \frac{\pi}{4} d_0^2 u_0 \Delta t. \qquad [2]$$

At low jet velocities, surface deformation of the jet is controlled by the most-unstable disturbance wave at both the early and latter stages of jet instability (Meister & Scheele 1969; Kitamura *et al.* 1982); i.e. higher harmonics are not prevalent during the transition process. Hence, it is assumed that each most-unstable disturbance wave produces a single droplet; so, $u_0 \Delta t = N\lambda_m$, where λ_m is the most-unstable wavelength. Thus, [2] becomes

$$\frac{\pi}{6} N d^3 = \frac{\pi}{4} d_0^2 N \lambda_m. \qquad [3]$$

Defining a dimensionless most-unstable wave number as $\eta_m \equiv \pi d_0/\lambda_m$, [3] may be rewritten as

$$\frac{d}{d_0} = \left(\frac{3\pi}{2\eta_m}\right)^{1/3}. \qquad [4]$$

Equation [4] provides an expression for droplet diameter in terms of η_m which, in turn, can be obtained from instability analysis. Several investigators (e.g. Tomotika 1935; Kinoshita *et al.* 1994) have analyzed jet instability under the conditions assumed in this study. Tomotika's dispersion equation can be expressed as

$$\det([a_{ij}]) = 0, \qquad [5]$$

where $[a_{ij}]$ is a 4×4 matrix. All of the elements in the matrix contain Bessel functions of η, η_1, or $\hat{\eta}$, where η is a dimensionless wave number, $\eta_1^2 \equiv \eta^2 + \beta/2Z$, $\hat{\eta}^2 \equiv \eta^2 + (\beta/2Z)\hat{\rho}/\rho\mu/\hat{\mu}$, $\beta \equiv \omega(2\rho r_j^3/\sigma)^{1/2}$ is a dimensionless growth rate, $Z \equiv \mu(2r_j\rho\sigma)^{-1/2}$ is the Ohnesorge number, and r_j is the undisturbed jet radius. (The caret signifies properties of the ambient fluid.) For low-velocity, liquid-in-liquid systems, it may be assumed that $r_j \approx d_0/2$. In concept, [5] may be expressed by the following relationship:

$$\beta = \beta(\eta, \eta_1, \hat{\eta}, \hat{\rho}/\rho, \hat{\mu}/\mu, Z).$$

η_m can be determined from this relationship; however, in general, [5] must be solved numerically (Meister & Scheele 1967; Kitamura & Takahashi 1986). Kinoshita *et al.* (1994) developed the following simple alternative to Tomotika's implicit, complex equation—an explicit dispersion equation applicable to both liquid-in-liquid and liquid-in-gas systems:

$$\left[1 + \frac{1}{2}\frac{\hat{\rho}}{\rho}\eta\frac{K_0(\eta)}{K_1(\eta)}\right]\beta^2 + 2Z\left(3 + \frac{\hat{\mu}}{\mu}\right)\eta^2\beta = \eta^2(1 - \eta^2), \quad [6]$$

where K_0 and K_1 are zeroth- and first-order modified Bessel functions of the second kind. The jet is unstable whenever $\beta > 0$, which requires $\eta < 1$. Under such conditions, $K_0(\eta)/K_1(\eta) < 1$ and $\eta K_0(\eta)/K_1(\eta) \ll 1$. Thus, for liquid jets that satisfy the condition, $\hat{\rho}/\rho < 2$ (which is generally met with liquid jets), [6] reduces to

$$\beta^2 + 2Z(3 + \hat{\mu}/\mu)\eta^2\beta = \eta^2(1 - \eta^2). \quad [7]$$

Based on Rayleigh's maximum instability theory, the most-unstable wave number can be obtained by applying the condition $\mathrm{d}\beta/\mathrm{d}\eta|_{\eta=\eta_m} = 0$ to [7]:

$$\eta_m = (2 + 2Z^*)^{-1/2}, \quad [8]$$

where $Z^* \equiv (3\mu + \hat{\mu})/(d_0\sigma\rho)^{1/2}$ is a modified Ohnesorge number. Substituting [8] into [4] and rearranging yields the following equation for droplet size

$$\frac{d}{d_0} = \left(\frac{3\pi}{\sqrt{2}}\right)^{1/3}(1 + Z^*)^{1/6}. \quad [9]$$

Equation [9] applies to low-velocity, liquid-in-liquid or liquid-in-gas systems, involving Newtonian or non-Newtonian fluids (provided that suitable apparent viscosities can be specified for the latter). For highly viscous liquid jets in low viscosity liquids, the effect of the ambient liquid becomes insignificant ($Z^* \to 3Z$) and [8] reduces to Weber's classical result (Weber 1931): $\eta_m = (2 + 6Z)^{-1/2}$. In this limiting case, Weber's instability theory can provide reasonably accurate droplet-size predictions.

3. COMPARISON OF THEORY WITH EXPERIMENT

To test the validity of the theoretical analysis described in the previous section, predictions of droplet size obtained using [9] are compared with experimental results for seventeen Newtonian and five non-Newtonian liquid-in-liquid systems (Kitamura *et al.* 1982; Kitamura & Takahashi 1982). All of the non-Newtonian liquids tested have shear stress versus deformation rate relationships that follow power laws (i.e. are ineleastic, shear-thinning viscous non-Newtonian fluids), having rheological parameters, K and n, that obey

$$\tau = K\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)^n. \quad [10]$$

3.1. Newtonian liquid systems

Kitamura *et al.* (1982) conducted experiments employing a variety of Newtonian fluids for the jet and ambient liquids. In their experiments, droplet images were captured with high-speed photography and the volume–surface mean diameters of 50–100 droplets were used to determine droplet size. In those tests, jets of either kerosene, *n*-heptane, *n*-butanol, or mixtures of liquid paraffin and kerosene discharged into water or aqueous solutions of starch syrup. The systems

Table 1. Newtonian systems: comparison of measured and predicted droplet sizes[a]

System[b]	Jet fluid (ambient fluid)[c]	Density ρ (kg/m^3)	Viscosity μ (10^{-3} Pa-s)	Interfacial tension σ (10^{-2} N/m)	Droplet size, d (cm)		
					Predicted via		
					Measured	[9]	[4] and [5]
1	n-heptane	676	0.403	5.12	0.237	0.230	0.234
	(water)	996	0.829				
2*	n-heptane	686	0.426	5.12	0.181	0.164	0.167
	(water)	999	1.09				
3	kerosene	790	1.27	4.42	0.227	0.230	0.235
	(water)	996	0.834				
4*	kerosene	797	1.40	4.47	0.165	0.164	0.167
	(water)	999	1.08				
5	kerosene/paraffin	831	8.00	4.22	0.240	0.234	0.237
	(water)	996	0.806				
6	kerosene/paraffin	848	18.8	4.04	0.250	0.239	0.241
	(water)	998	1.03				
7*	kerosene/paraffin	818	3.34	4.27	0.160	0.165	0.168
	(water)	999	1.05				
8	n-heptane	683	0.407	5.60	0.183	0.230	0.168
	(aq.SS sol.)	1126	3.71				
9	kerosene	788	1.25	4.50	0.244	0.231	0.235
	(aq.SS sol.)	1117	2.73				
10	kerosene	790	1.26	4.95	0.249	0.231	0.236
	(aq.SS sol.)	1154	5.46				
11	kerosene/paraffin	831	8.00	4.30	0.255	0.234	0.238
	(aq.SS sol.)	1117	2.78				
12	kerosene/paraffin	847	15.9	4.20	0.260	0.238	0.240
	(aq.SS sol.)	1116	2.61				
13	kerosene/paraffin	849	17.3	3.90	0.268	0.239	0.242
	(aq.SS. sol.)	1132	3.35				
14	kerosene/paraffin	854	24.7	2.52	0.278	0.245	0.247
	(aq.SS.sol.)	1158	6.97				
15	kerosene/paraffin	848	18.1	4.20	0.283	0.242	0.244
	(aq.SS. sol.)	1233	22.9				
16*	kerosene/paraffin	817	3.30	4.75	0.202	0.165	0.169
	(aq.SS. sol.)	1126	3.80				
17*	n-butanol	849	3.87	1.50	0.180	0.173	0.175
	(water)	991	1.33				

[a]Properties information and experimental data from Kitamura *et al.* (1982).
[b]* denotes systems with $d_0 = 0.087$ cm; $d_0 = 0.122$ cm for all other systems.
[c]SS denotes starch syrup.

examined fall within the following ranges of property values: $0.61 < \hat{\rho}/\rho < 1.47$, $0.05 < \hat{\mu}/\mu < 9.12$, and $1.88 \times 10^{-3} < Z < 152 \times 10^{-3}$. Table 1 provides details of the experiments and compares measured droplet sizes with the predictions of the present study. The relationship between droplet size (both measured and predicted) and modified Ohnesorge number is shown in figure 1.

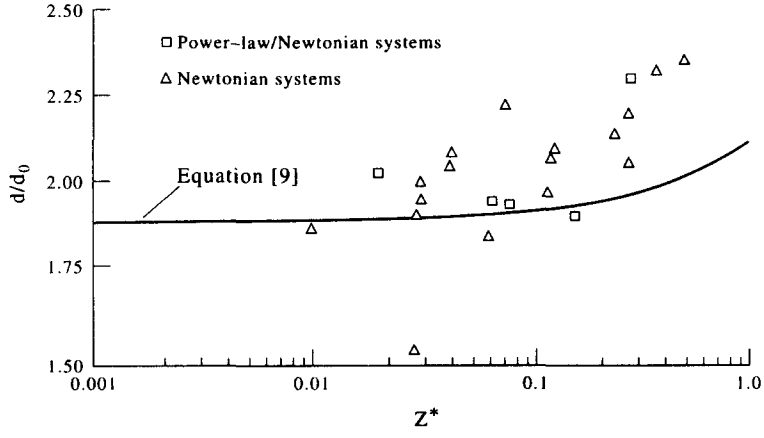

Figure 1. $d/d_0 = f(Z^*)$ relationship. Experimental data: Newtonian systems from Kitamura *et al.* (1982); power-law/Newtonian systems from Kitamura and Takahashi (1982).

3.2. *Power-law liquid/Newtonian liquid systems*

Two different situations involving power-law non-Newtonian liquids are analyzed: (1) Newtonian liquid jets discharging into power-law liquids; and (2) power-law liquid jets discharging into Newtonian liquids. In the former, the jet fluid is either *n*-heptane or mixtures of liquid paraffin and kerosene, and the ambient fluid comprises aqueous carboxymethyl cellulose (CMC) solutions; in the latter, the jet fluid is either polystyrene or styrene butadiene rubber (SBR) solutions in xylene, and the ambient fluid is water. In the experiments, droplet sizes were determined via the same method as that for Newtonian systems.

For power-law liquid jets in Newtonian liquids, at the jet/ambient fluid interface, [10] may be rewritten as

$$\tau_{rzs} = K\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)_s^{n-1}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)_s, \qquad [11]$$

where the subscript s denotes the conditions at the interface. For wave motions in which wavelength $\lambda \gg r_j$, $|\partial u_z/\partial r|_s \gg |\partial u_r/\partial z|_s$ (Levich 1962); thus, the term on the right-hand side of [11] containing rheological parameters may be approximated as

$$K\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)_s^{n-1} \approx K\left(\frac{\partial u_z}{\partial r}\right)_s^{n-1}. \qquad [12]$$

For low velocity liquid jets, it may be assumed that $(\partial u_z/\partial r)_s \sim u_0/r_j \sim u_0/(d_0/2)$; thus, $K(\partial u_z/\partial r)_s^{n-1} = K'(d_0/u_0)^{1-n}$. Equation [11] becomes

$$\tau_{rzs} = K'\left(\frac{d_0}{u_0}\right)^{1-n}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)_s. \qquad [13]$$

An apparent viscosity for the power-law jet liquid can be defined as $\mu^* \equiv K'(d_0/u_0)^{1-n}$. In temporal instability, the liquid jet essentially is a capillary tube; therefore, a general expression for Reynolds number is (Skelland 1967)

$$\mathrm{R_e} = \frac{d_0^n u_0^{2-n}\rho}{8^{n-1}K}\left(\frac{4n}{3n+1}\right)^n. \qquad [14]$$

Comparing [14] with the equivalent relationship for Newtonian fluids yields the following expression for the apparent viscosity of the jet liquid

$$\mu^* = \frac{K}{8}\left(6 + \frac{2}{n}\right)^n\left(\frac{d_0}{u_0}\right)^{1-n}. \qquad [15]$$

Thus, $K' = (K/8)(6 + 2/n)^n$.

For Newtonian liquid jets in power-law liquids,

$$\hat{\tau}_{rzs} = \hat{K}\left(\frac{\partial \hat{u}_r}{\partial z} + \frac{\partial \hat{u}_z}{\partial r}\right)_s^{\hat{n}-1}\left(\frac{\partial \hat{u}_r}{\partial z} + \frac{\partial \hat{u}_z}{\partial r}\right)_s. \qquad [16]$$

If a coordinate system that moves with the jet is chosen, near the interface, the flow field of the ambient fluid is primarily induced by the surface disturbance wave; thus, for capillary instability, $|\partial \hat{u}_z/\partial r|_s \gg |\partial \hat{u}_r/\partial z|_s$ (Kinoshita *et al.* 1994). The term on the right-hand side of [16] containing rheological parameters can be approximated as

$$\hat{K}\left(\frac{\partial \hat{u}_r}{\partial z} + \frac{\partial \hat{u}_z}{\partial r}\right)_s^{\hat{n}-1} \approx \hat{K}\left(\frac{\partial \hat{u}_z}{\partial r}\right)_s^{\hat{n}-1}. \qquad [17]$$

In the ambient liquid, viscous effects are important only near the interface; i.e. in the induced flow region (Kinoshita *et al.* 1994). If the viscous layer is assumed to be confined to the region $r_j < r < r_j + d_0$, then $(\partial \hat{u}_z/\partial r)_s \sim u_0/d_0$; therefore,

$$\hat{\tau}_{rzs} = \hat{K}'\left(\frac{d_0}{u_0}\right)^{1-\hat{n}}\left(\frac{\partial \hat{u}_r}{\partial z} + \frac{\partial \hat{u}_z}{\partial r}\right)_s. \qquad [18]$$

As done earlier, the apparent viscosity for the power-law ambient liquid may be defined as $\hat{\mu}^* \equiv \hat{K}'(d_0/u_0)^{1-\hat{n}}$. Since K' depends only on rheological parameters, it may be assumed that

Table 2. Non-Newtonian systems: comparison of measured and predicted droplet sizes[a]

System[b]	Jet fluid (ambient fluid)	Density ρ (kg/m^3)	Viscosity μ (10^{-3} Pa-s)	Interfacial tension σ (10^{-2} N/m)	Droplet size, d (cm) Measured[c]	Predicted via [9]
1*	n-heptane	687	0.370	5.02	0.176	0.164
	(0.25 wt% aq. CMC sol. $K/n = 0.028/0.71$)	1000	—			
2*	kerosene/paraffin	816	3.23	4.27	0.168	0.166
	(0.12 wt% aq. CMC sol. $K/n = 0.0106/0.88$)	1000	—			
3*	kerosene/paraffin	842	14.2	4.00	0.200	0.171
	(0.18 wt% aq. CMC sol. $K/n = 0.0159/0.88$)	1000	—			
4	polystyrene/xylene $K/n = 0.0088/0.88$	878	—	2.58	0.237	0.232
	(water)	999	1.16			
5	SBR/xylene $K/n = 0.0223/0.88$	874	—	2.58	0.232	0.230
	(water)	999	1.19			

[a]Properties information and experimental data from Kitamura & Takahashi (1982).
[b]* denotes systems with $d_0 = 0.087$ cm; $d_0 = 0.122$ cm for all other systems.
[c]Obtained by interpolating graphical results from Kitamura & Takahashi (1982).

$\hat{K}' = (\hat{K}/8)(6 + 2/\hat{n})^{\hat{n}}$; thus, [15] also applies to the power-law ambient liquid. Therefore, for power-law/Newtonian systems, the modified Ohnesorge number can be evaluated by employing [15].

Table 2 provides details of experiments performed by Kitamura & Takahashi (1982), and compares droplet diameters measured in that study with predictions based on [9]. The relationship, $d/d_0 = f(Z^*)$, for power-law/Newtonian liquid systems is also shown in figure 1. In the experiments, $0 < u_0 < 1.2$ m/s. Average apparent viscosities, obtained for reference velocities of $u_0 = 0.4$, 0.8, and 1.2 m/s, were used to determine the modified Ohnesorge number.

4. DISCUSSION

Tables 1 and 2 (and figure 1) indicate good agreement between droplet diameters predicted by this study and measured values for both Newtonian and power-law/Newtonian systems. For Newtonian systems, the average difference between predictions and measurements is 8.1% and the maximum difference is 25.7%; for power-law/Newtonian systems, the average difference is 5.1% and the maximum difference is 14.5%. In the five power-law/Newtonian systems studied, the "viscous effects" of power-law fluids on droplet sizes were described well by utilizing apparent viscosities. In both Newtonian and power-law/Newtonian systems, the largest differences in predicted and measured behavior occur in systems comprising fluids with high viscosities. These differences may reflect a significant orifice effect which is not considered in the instability analysis based on an infinitely long, cylindrical liquid jet. (Based on the empirical correlation introduced by Kitamura & Takahashi (1986), $r_j/(d_0/2) = 1 + f(B_0)$, where $\beta_0 \equiv d_0^2 g \Delta\rho/\sigma$ is the Bond number. For all of the systems investigated, $f(B_0) < 0.05$; thus, $r_j \approx d_0/2$. Therefore, the influence of Bond number is negligible, and the differences are believed to be due primarily to the orifice effect.)

Density ratio does not appear to have a pronounced effect on droplet size in the seventeen Newtonian and five power-law/Newtonian systems examined in the previous section. This is consistent with the previous analysis of Kinoshita *et al.* (1994).

Inspection of [8] and [9] reveals that both the most-unstable wave number and droplet size can be written solely in terms of the modified Ohnesorge number. The modified Ohnesorge number can be interpreted as a ratio of the effective viscous force to the interfacial tension. Based on [9], droplet size increases as the effective viscosity increases or the interfacial tension decreases. Equation [9] has two limiting cases. When $Z^* \ll 1$, $d = 1.88 d_0$. This result agrees well with Tyler's experimental data for water jets in air (Tyler 1933), for which $d = 1.91 d_0$ (for water jets in air, $d_j \approx d_0$ (Tyler 1933)). Hence, for inviscid systems, the droplet size is independent of fluid properties. When $Z^* \gg 1$,

$d = 1.88\, d_0 Z^{*1/6}$. This indicates that for highly viscous systems, droplet size is much larger than that predicted by Tyler's relationship.

Concerning the orifice effect, [9] may be expressed more generally as

$$\frac{d}{d_0} = 1.88\alpha(1 + Z^*)^{1/6}, \qquad [19]$$

where α is an experimentally determined orifice parameter representing the effect of the initial disturbance. According to Grant & Middleman (1966), the initial disturbance depends only on Ohnesorge number, Z, for liquid-in-gas systems. For liquid-in-gas systems, $Z^* \to 3Z$; therefore, in the present study, it is assumed that $\alpha = \alpha(Z^*)$ for liquid-in-liquid systems. Based on experimental data presented in tables 1 and 2 ($0.01 \leqslant Z^* \leqslant 0.50$), an empirical orifice parameter is determined to be

$$\alpha = 1.1303 + 0.0236 \ln Z^*. \qquad [20]$$

The relationship between d/d_0 and Z^* based on [19] and [20] is shown in figure 2. Indeed, including α into [9] significantly improves its accuracy at large modified Ohnesorge numbers (i.e. high viscosity systems). Excluding the single data point at $Z^* = 0.029$ (system 8 in table 1), for 21 liquid-in-liquid systems, the average difference between predicted and measured droplet diameters is 3.92% and the maximum difference is only 10.5%. Since the viscosities of the jet and the ambient liquid are proportional to the modified Ohnesorge number, in general, increasing viscosity leads to larger droplets. The density of the jet fluid and the interfacial tension both are in the denominator of Z^*; therefore, denser jet fluids and larger interfacial tension tend to form smaller droplets.

As an alternative to the present approach, [4] and [5] can be combined to predict droplet size; however, [5] must be solved numerically. Droplet sizes predicted from [4] and [5] for Newtonian systems (Kitamura *et al.* 1982) are also presented in table 1. Good agreement between the droplet sizes predicted in this study and those using [4] and numerical solutions to [5] are observed in table 1. Excluding system 8, the maximum difference between the two alternative droplet size predictions is only 2.4%. (For system 8, the value obtained from [4] and [5] is suspect, since the viscosity ratio for system 8 is 29 times that of system 7, which should result in larger droplets, as confirmed by experiment.)

As mentioned in the section "Theory", [6] and Tomotika's equation, [5], were obtained assuming the same conditions. Kinoshita *et al.* (1994) have shown that by applying appropriate conditions, [6] reduces to all published limiting solutions to Tomotika's equation. The systems presented in Table 1 do not belong to any of those limiting cases. The comparisons in table 1 demonstrate that [6] (and its reduced form, [7]) agrees well with Tomotika's equation in non-limiting cases as well. This suggests that, in general, predictions of the instability of liquid jets provided by [6] closely match those obtained from Tomotika's equation for the same conditions. However, in practice, [6] obviously is much simpler to use than Tomotika's equation, [5].

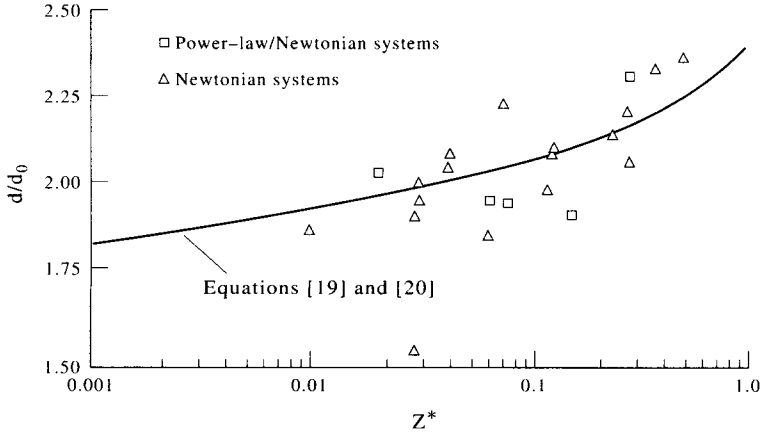

Figure 2. $d/d_0 = f(Z^*)$ relationship based on [19] and [20].

5. CONCLUSIONS

Based on a model of uniform-size droplet formation, and employing the instability analysis developed previously by the present authors, a simple equation is obtained for the size of droplets formed during the breakup of cylindrical liquid jets. This droplet-size equation applies to low-velocity liquid jets, involving Newtonian or power-law non-Newtonian fluids.

Predictions of droplet diameter from this study were compared with experimental data taken from the literature. Good agreement was observed—for seventeen Newtonian and five power-law/Newtonian liquid-in-liquid systems, the average differences are 8.1 and 5.1%, respectively; for water jets in air (liquid-in-gas system), the difference is only 1.6%.

The droplet diameter predictions from this study were also compared with those obtained from numerical solutions to Tomotika's equation for the seventeen Newtonian systems. Excellent agreement was observed. This suggests that the dispersion equation obtained previously by the present authors, [6], provides a simple alternative to Tomotika's complicated dispersion relationship.

Neglecting the density ratio did not cause any substantial difference in the results for the 23 liquid-in-liquid systems investigated (both Newtonian and power-law/Newtonian systems). For the five power-law/Newtonian systems, the effect of viscosity on droplet size was well accommodated by utilizing the apparent viscosity. In addition, it was demonstrated that droplet size is controlled by a single parameter, the modified Ohnesorge number.

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